TORRICELLI'S DISCHARGE LAW MODIFIED BY THE BATHTUB VORTEX

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Introduction

Experimental set-up

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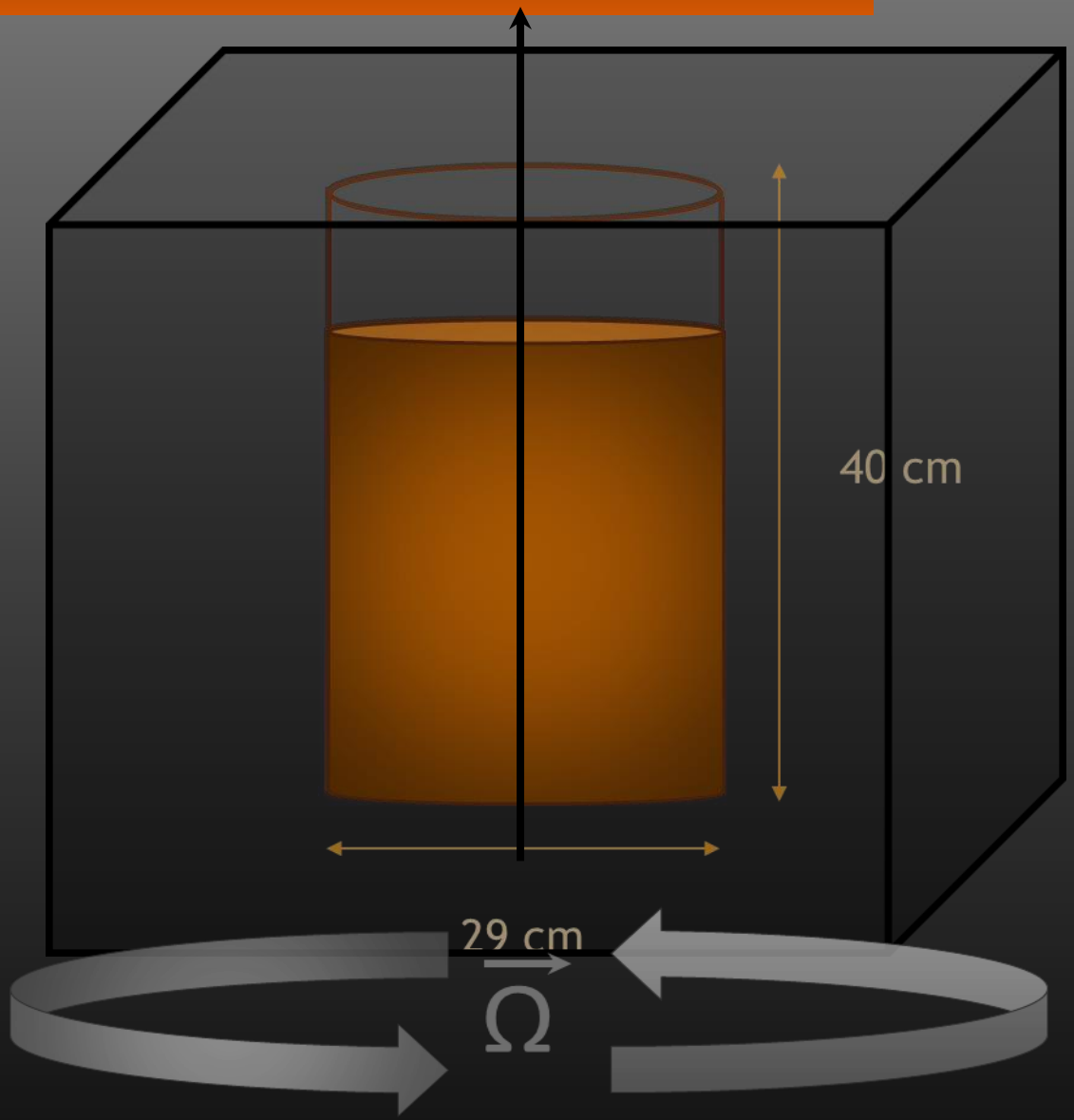
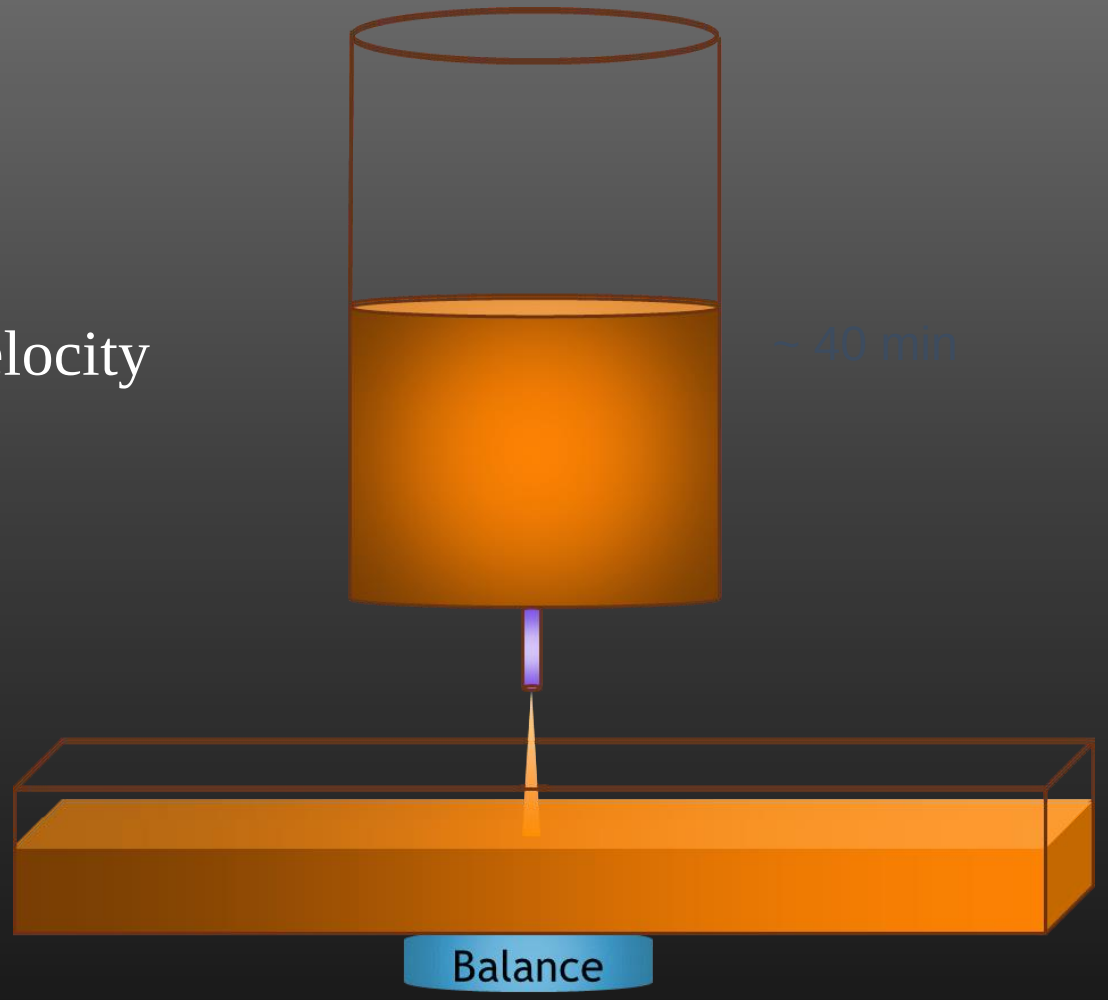


Mass measurements

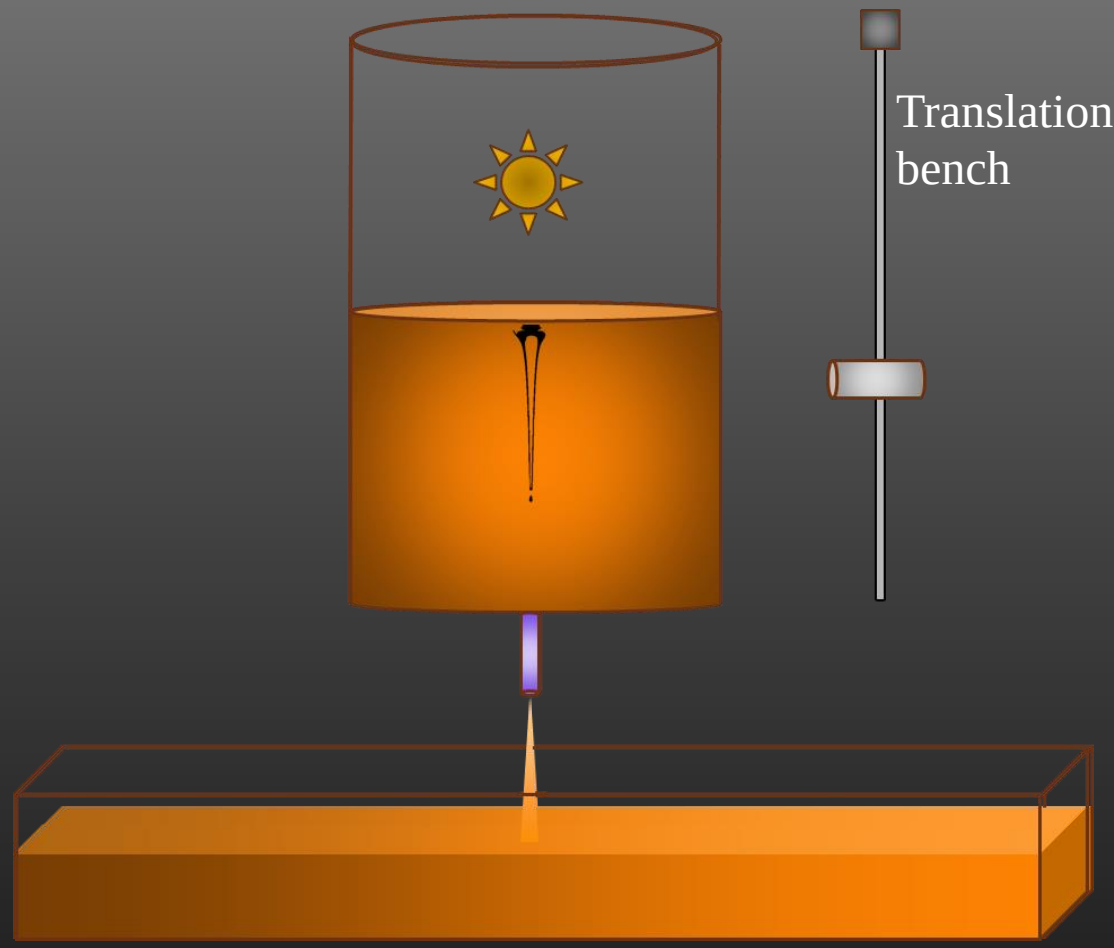
Mass measurements every 2s

- Mass -> Level of water
- Derivative of mass-> Draining velocity



Shape of the interface

Photo every 10s



Particule Image Velocimetry

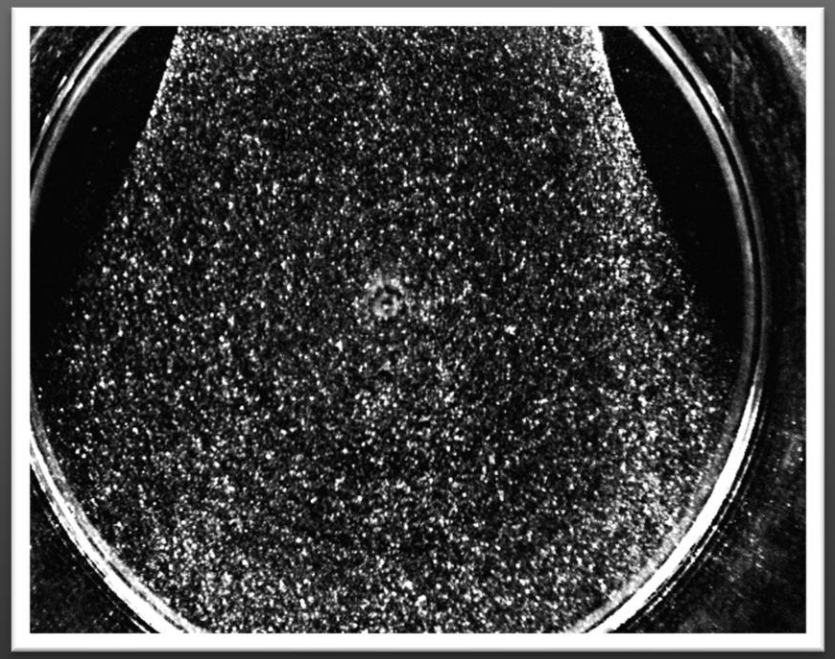
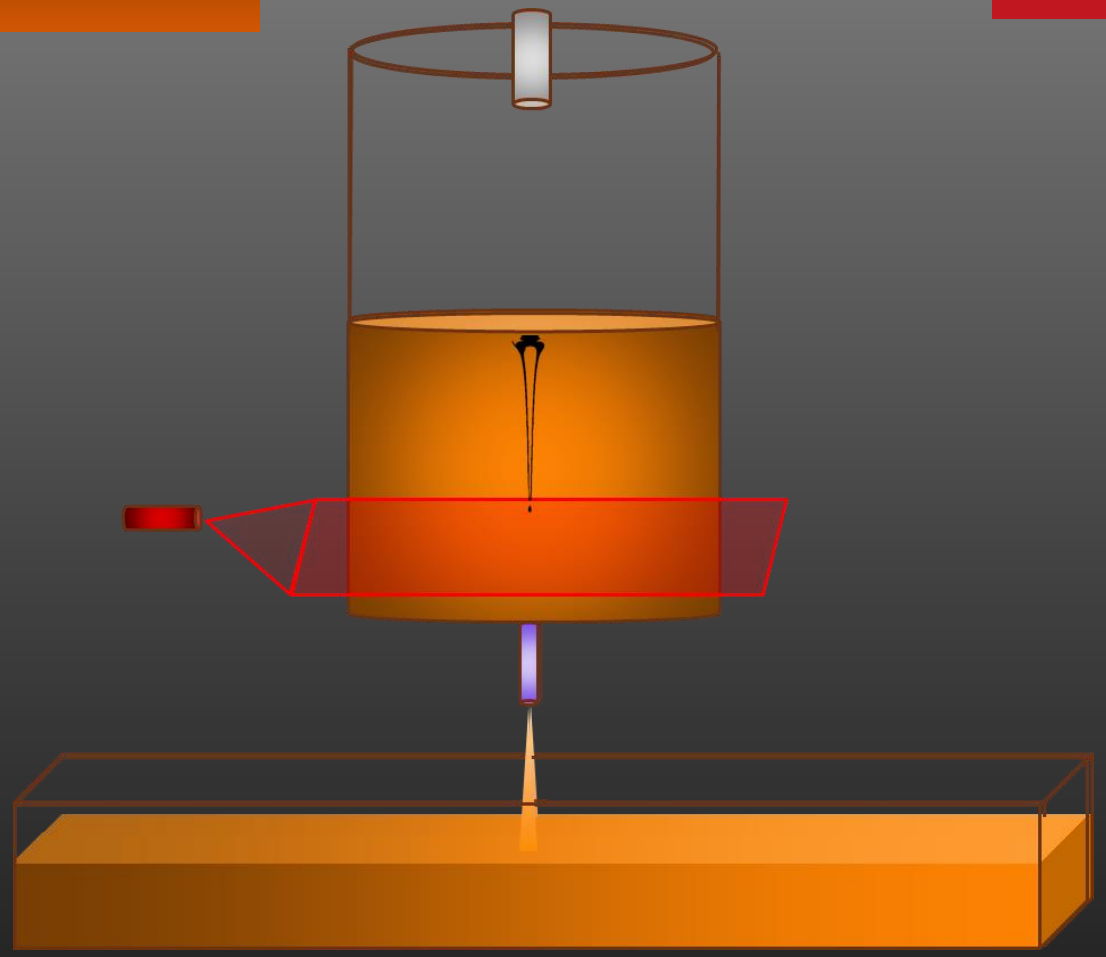
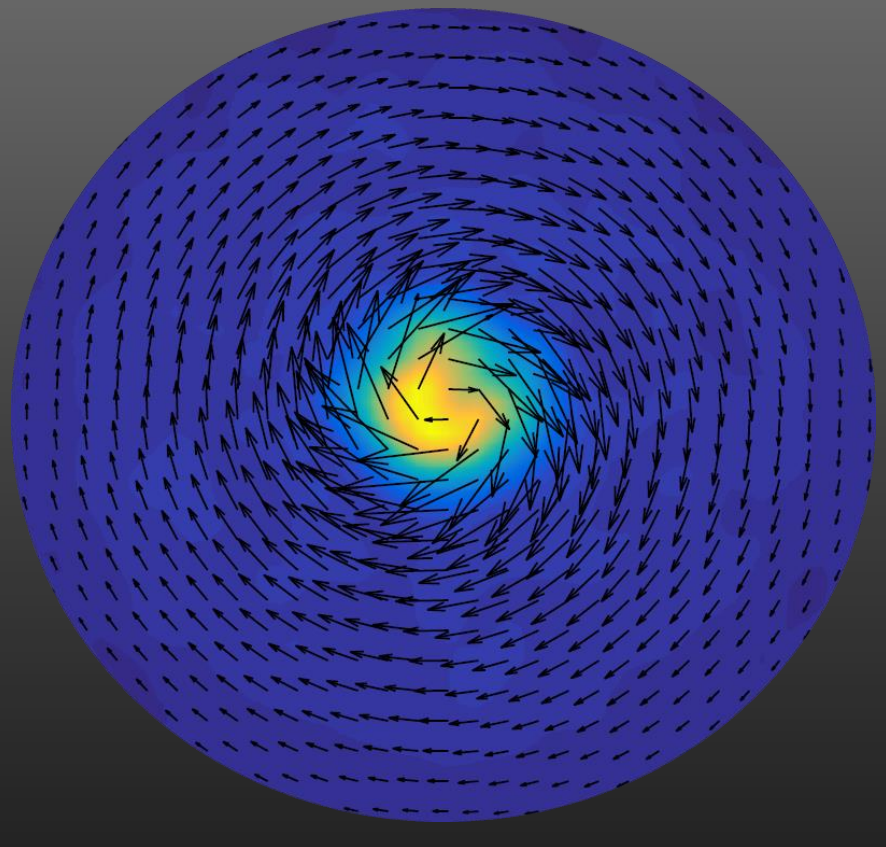


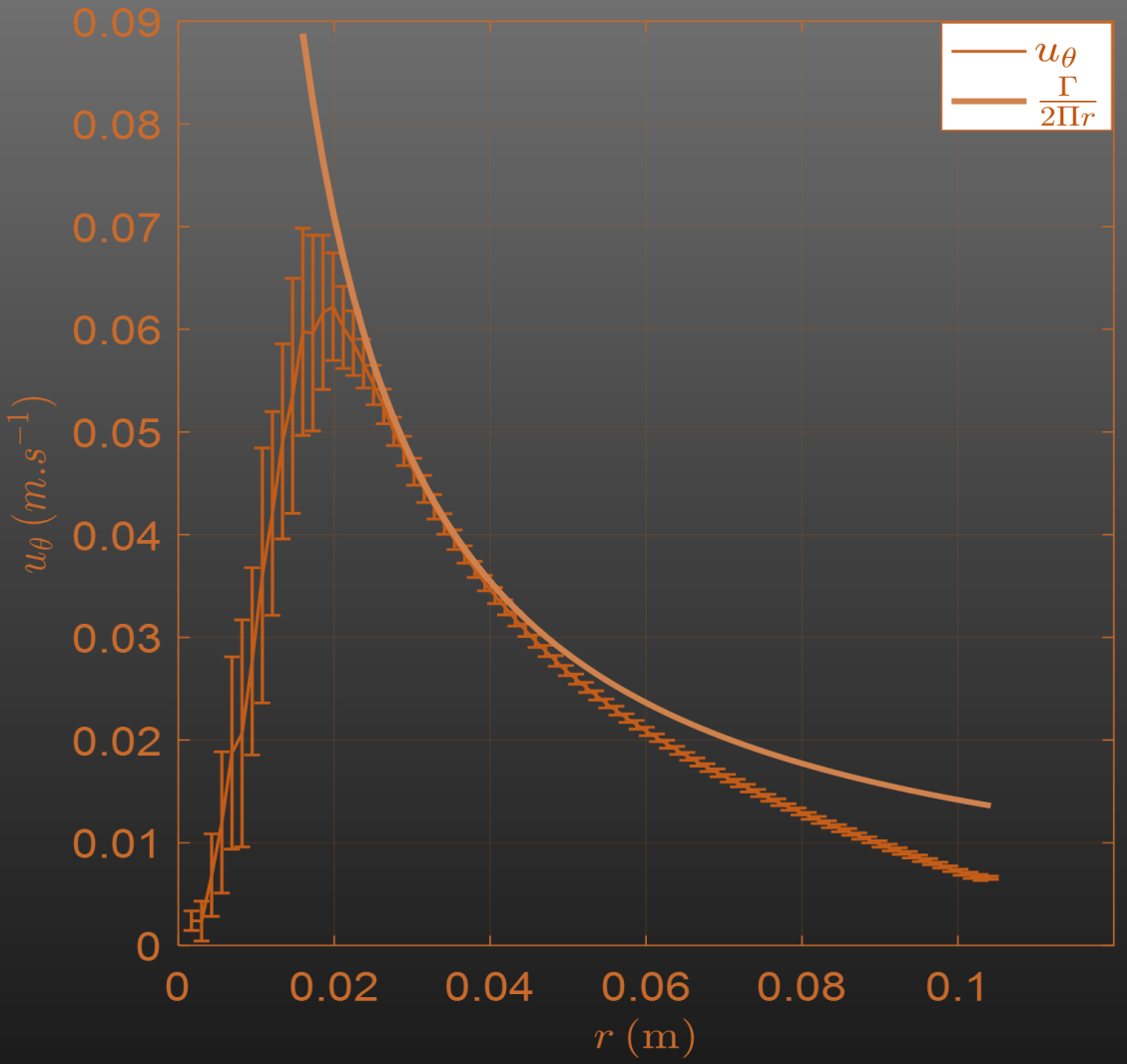
Photo every 10s



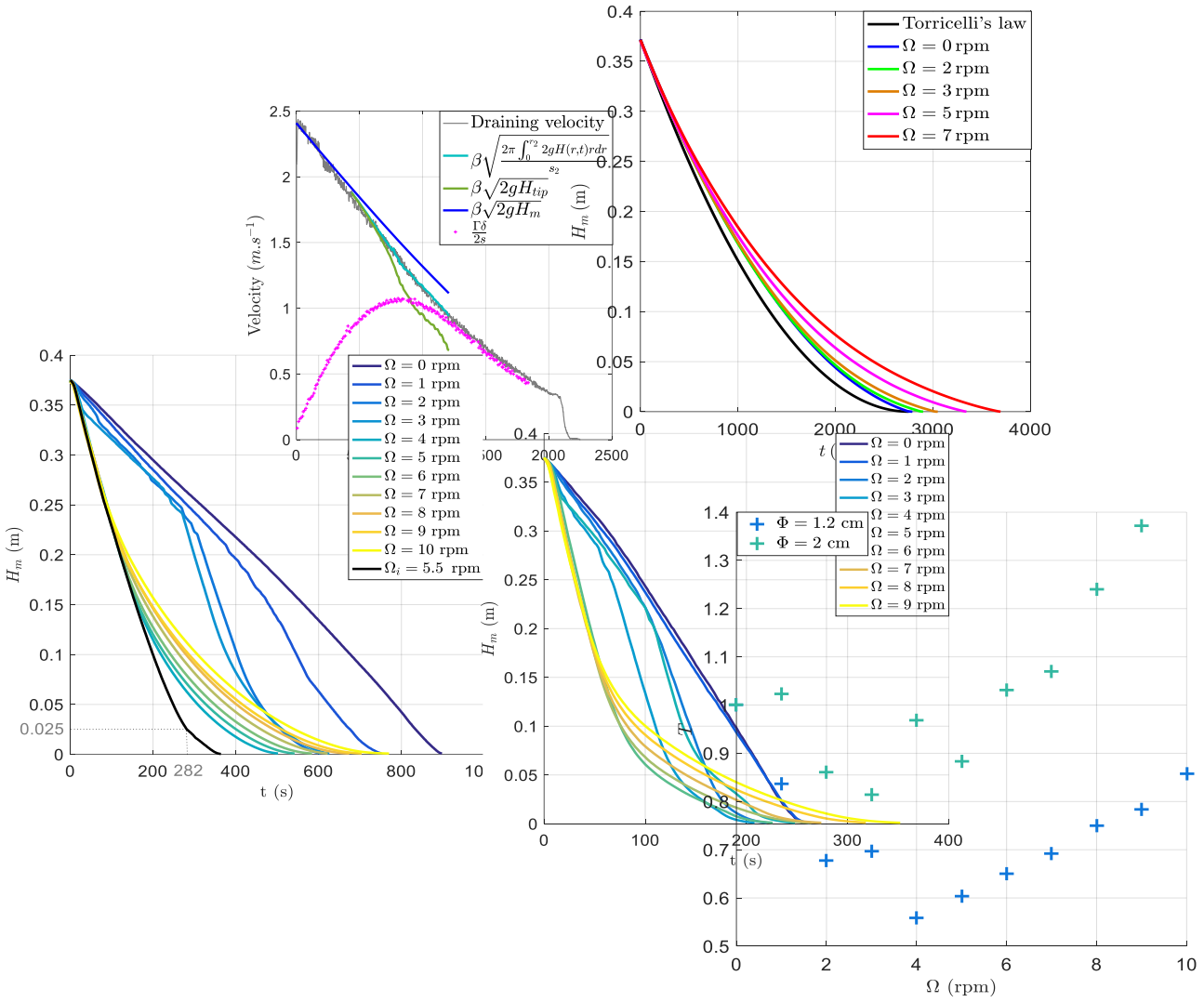
➡ Circulation of the vortex

Particule Image Velocimetry



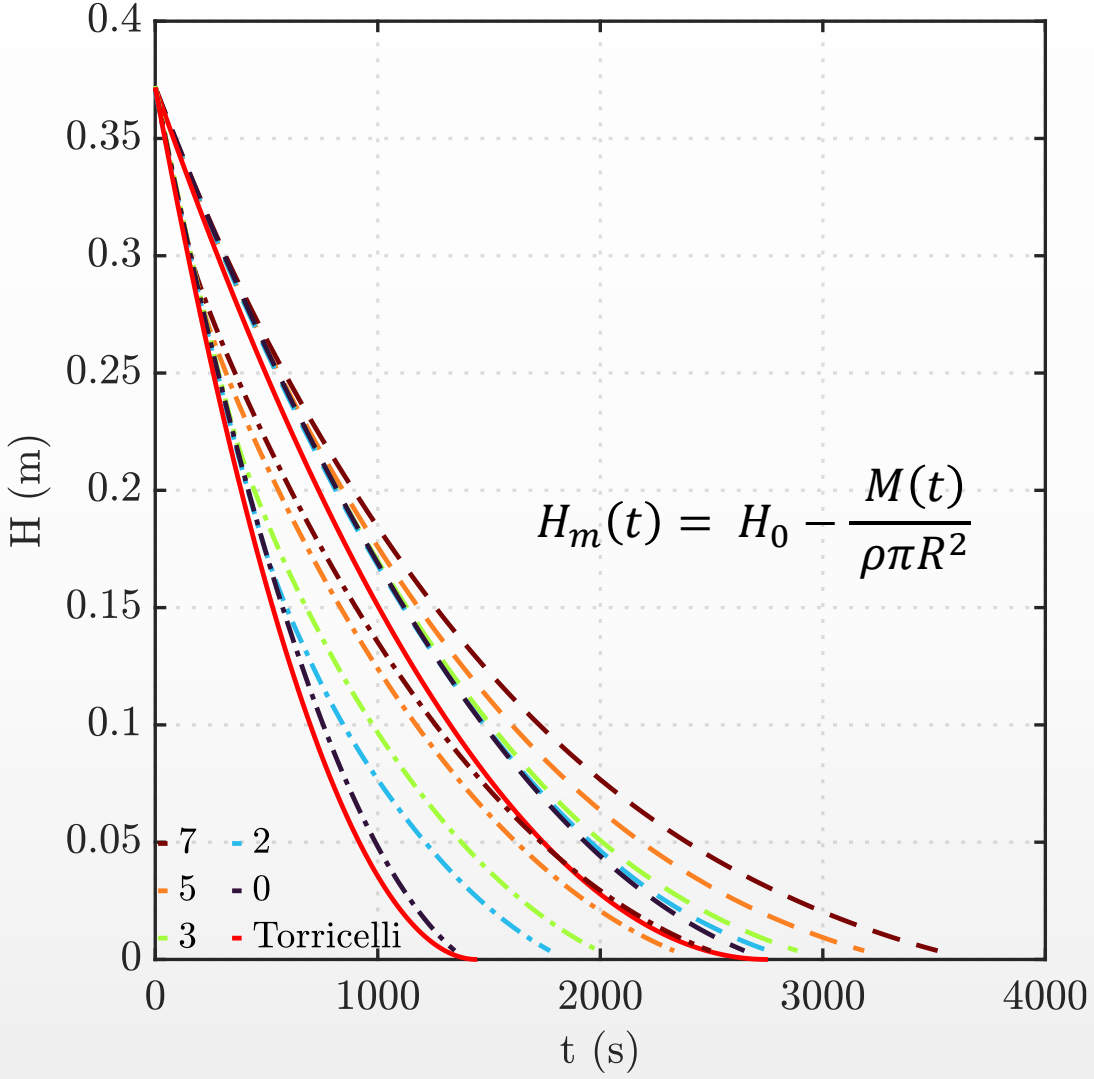


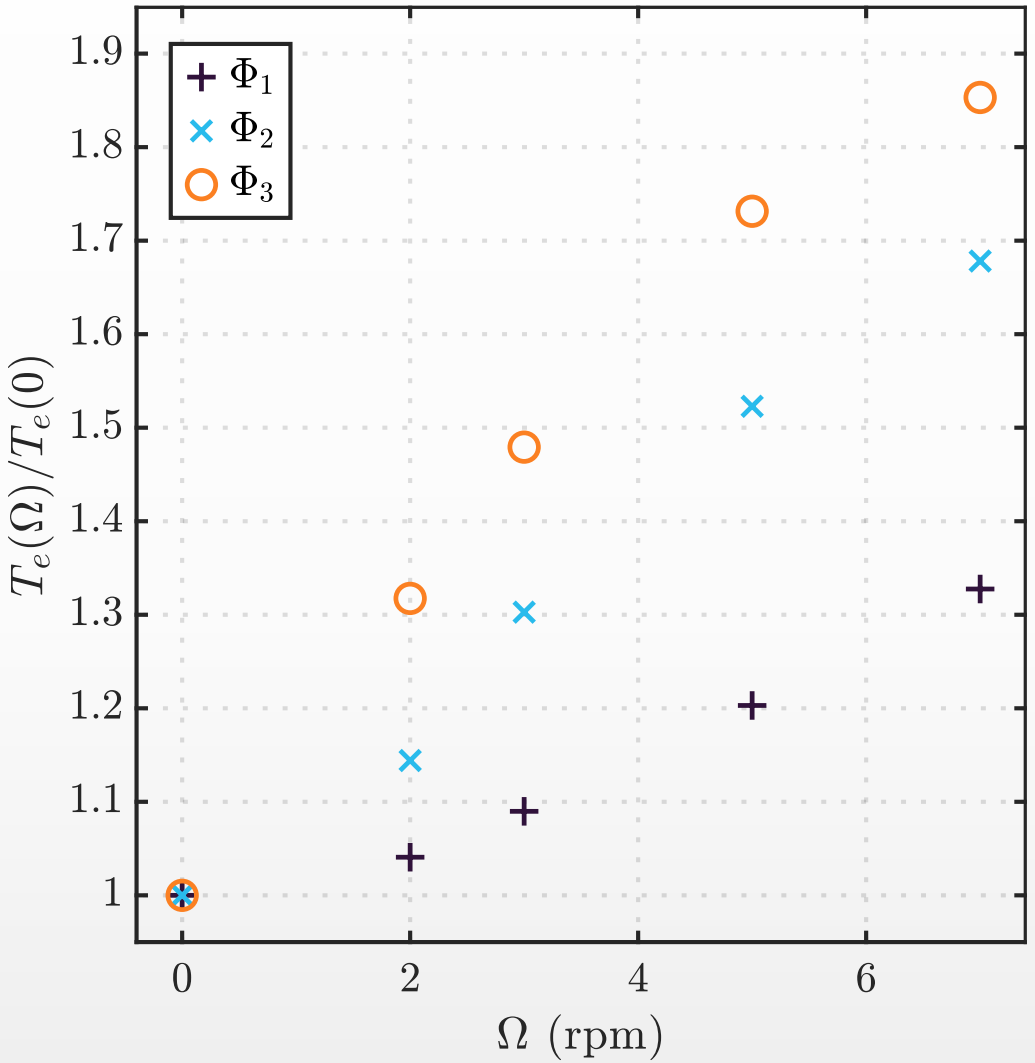
Experimental results



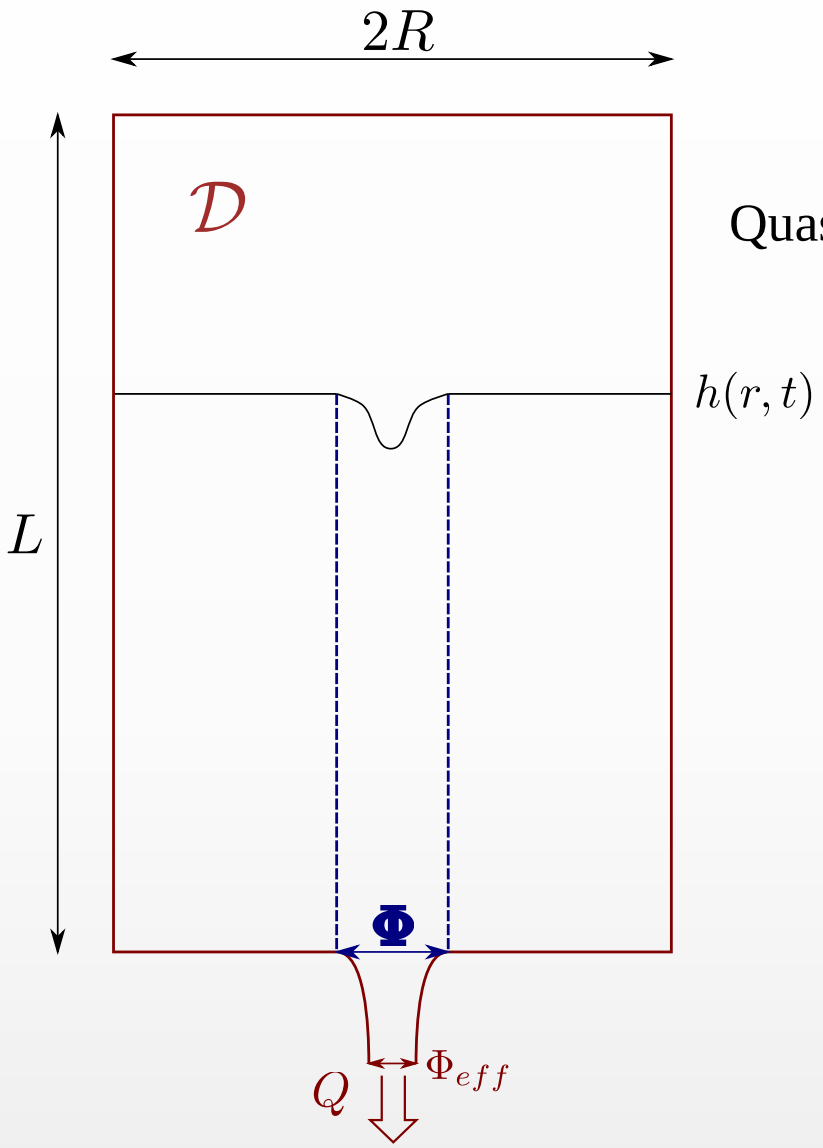
Evolution of the water height

$\mathbf{\Phi_1} = 3$mm (dash dotted line), $\mathbf{\Phi_2} = 3{,}5$mm, $\mathbf{\Phi_3} = 4$mm (dash line)





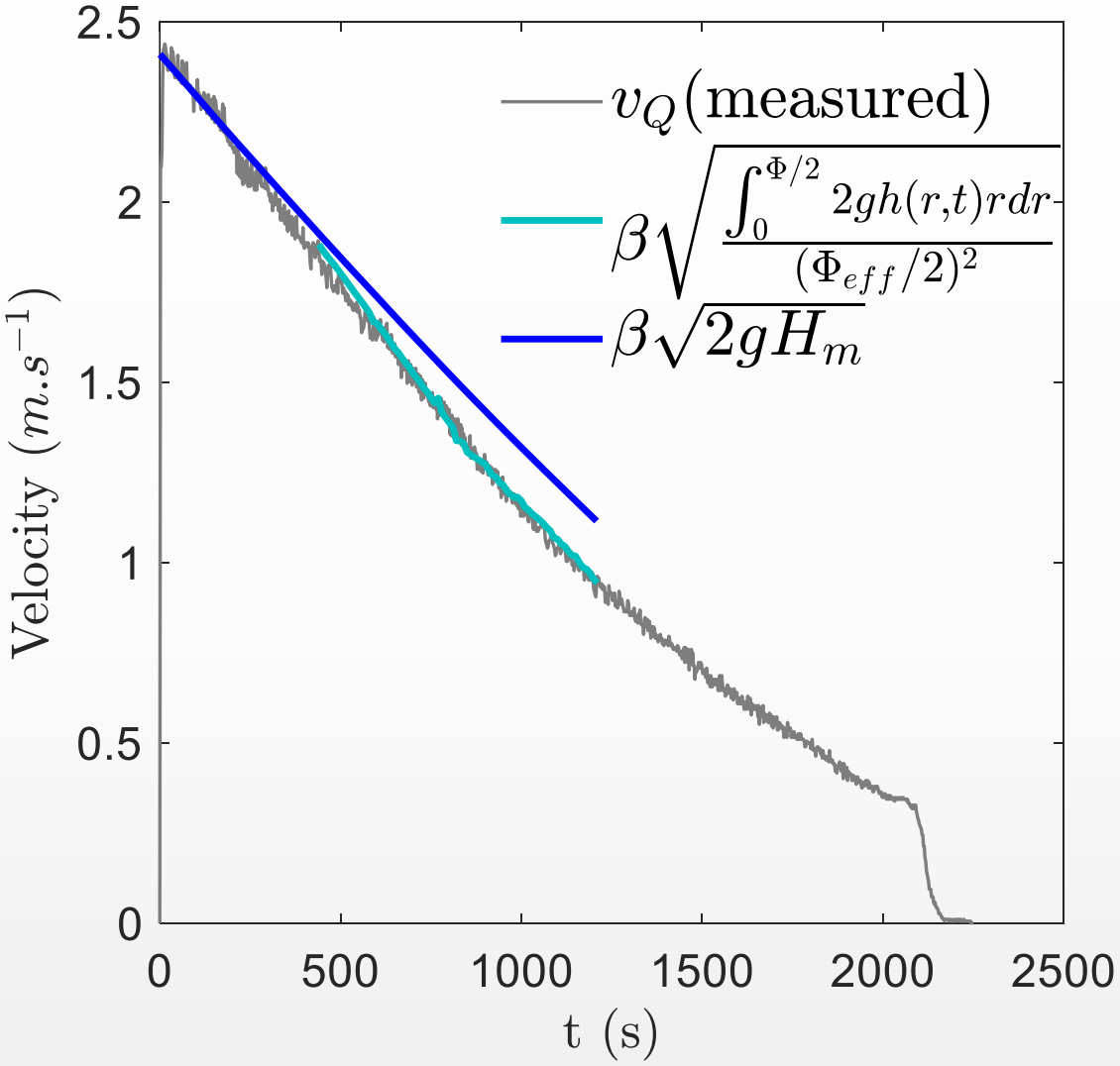
Momentum conservation law



Quasi-static hypothesis + vena contracta phenomenon:

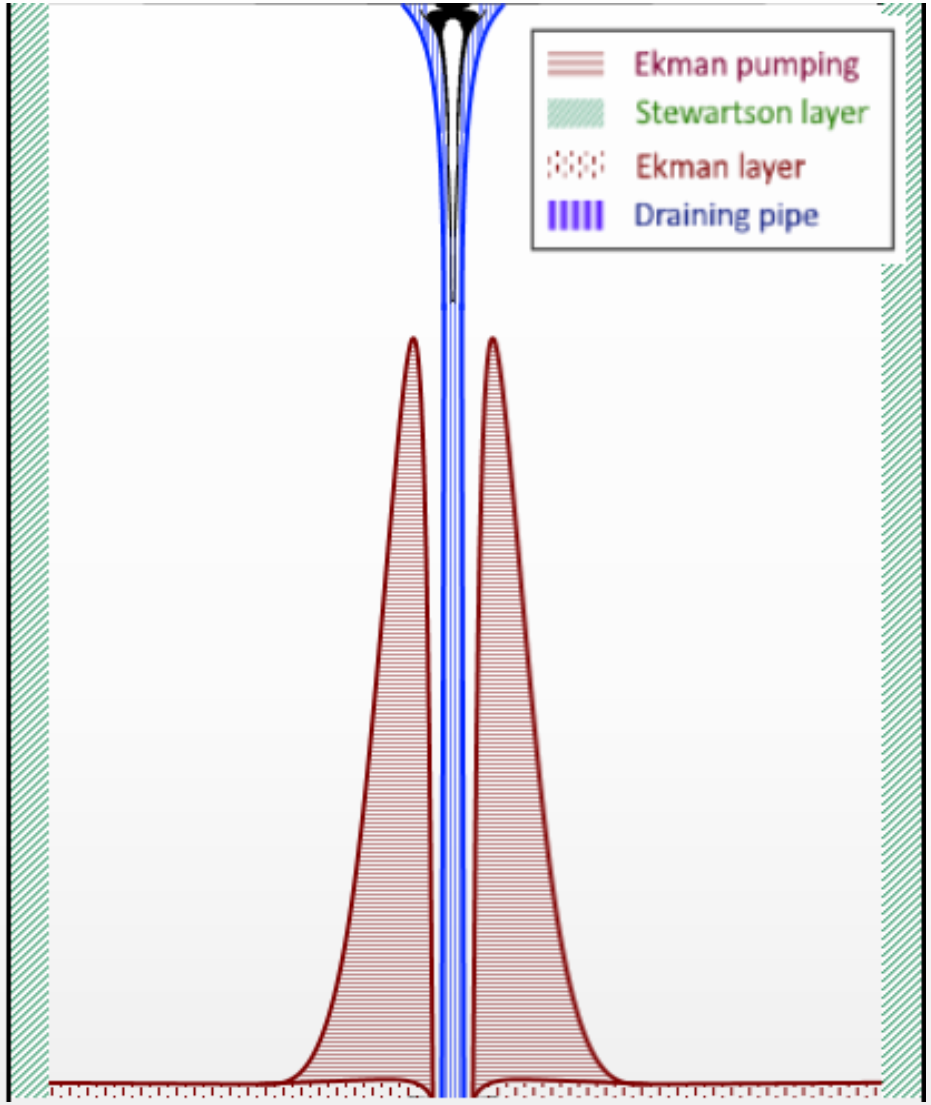
$$v_s(t) = \sqrt{\frac{2g}{\Phi_{\text{eff}}^2} \int_0^{\Phi} h(r,t) r dr}$$

Draining velocity

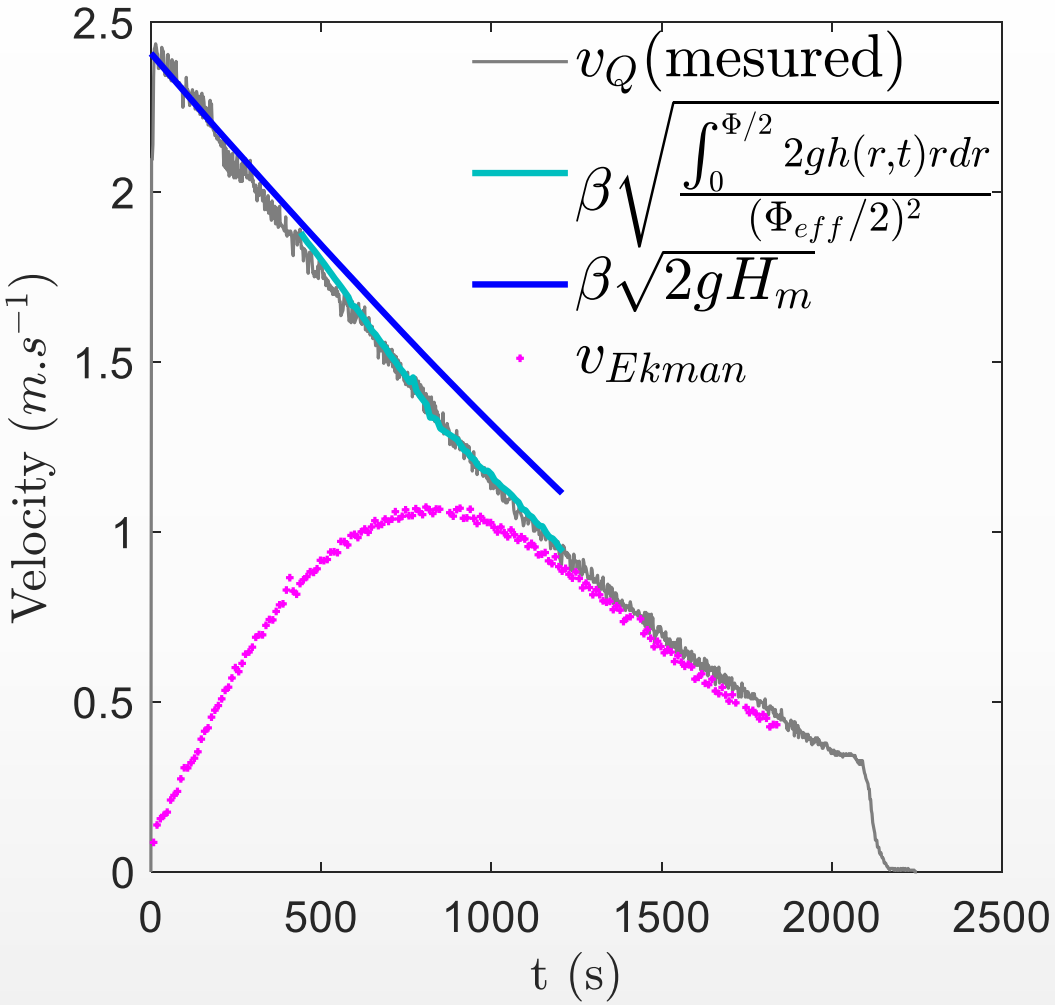


$\beta \sim 0.9$ correction coefficient due to vena contracta phenomenon.

Flow structure



Draining velocity



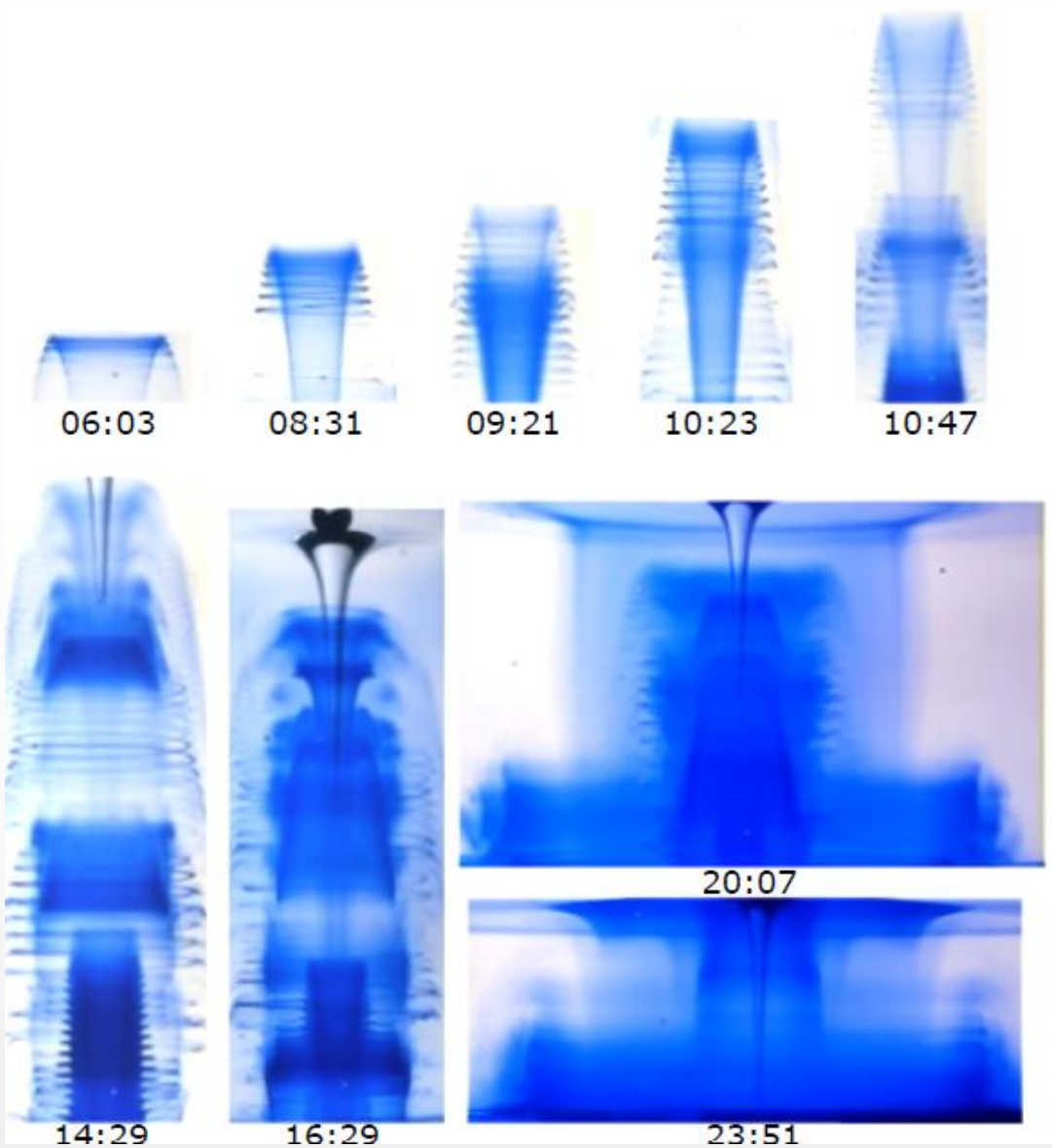
Ekman boundary layer theory + the Rankine model in the bulk.

F the flow through Ekman layer:

$$F = v_{Ekman} s_2 = \int_0^{\infty} 2\pi r \widetilde{u_r} dz = \delta\Gamma/2$$

with the Ekman length : $\delta = \sqrt{\nu/\Omega}$

Ekman pumping



Conclusions

We obtain a flow law for vortex emptying by expressing the momentum conservation with an experimental validation.

We also noticed that a large part of the drained flow passed through the Ekman boundary layers at the end of the draining process.