Journée Dynamique des Fluides du Plateau de Saclay 2022 – 18/03/2022

Extreme waves in coastal areas: assessment of nonlinear effects induced by an abrupt variation of water depth

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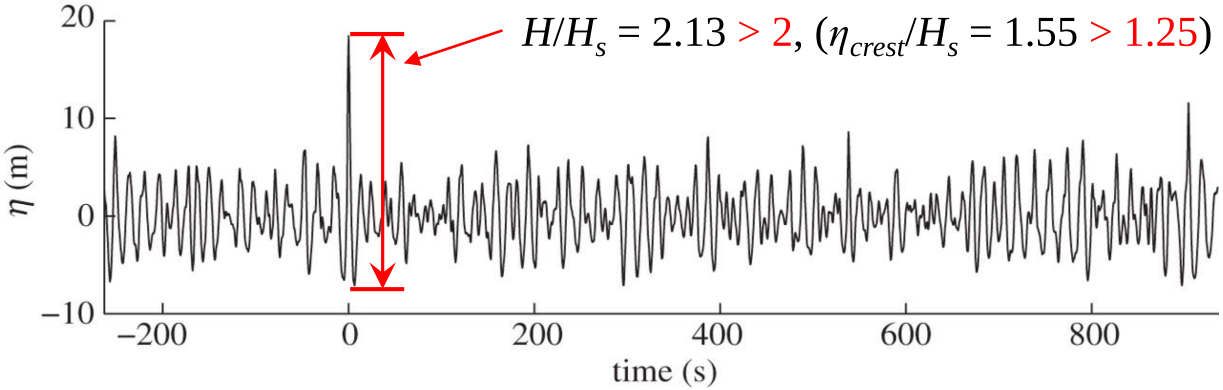
Introduction – Freak wave characteristics

Definition / criterion

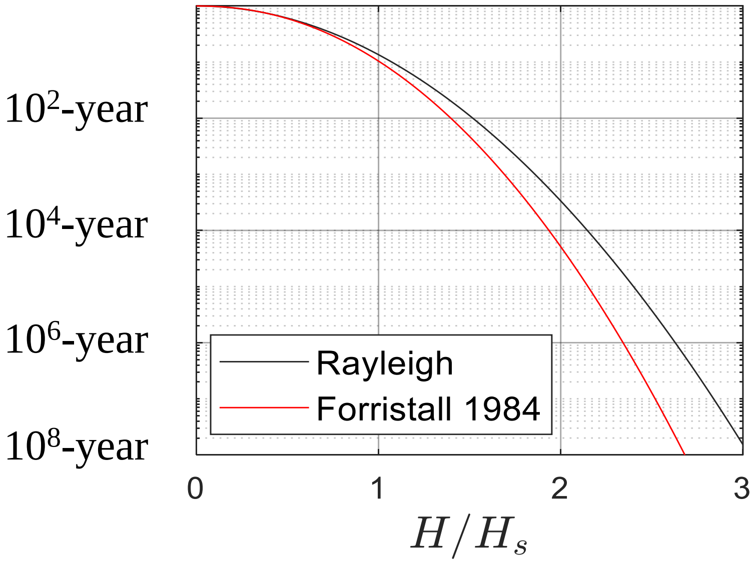
$$\frac{\boldsymbol{H}}{\boldsymbol{H_s}} > \mathbf{2} \ \text{ or } \ \frac{\eta_{crest}}{H_s} > 1.25$$

(H_s: significant wave height)



New Year wave (1st January 1995) at Draupner platform

Return Period – Wave height



Introduction – Freak waves mechanisms

Quasi-linear

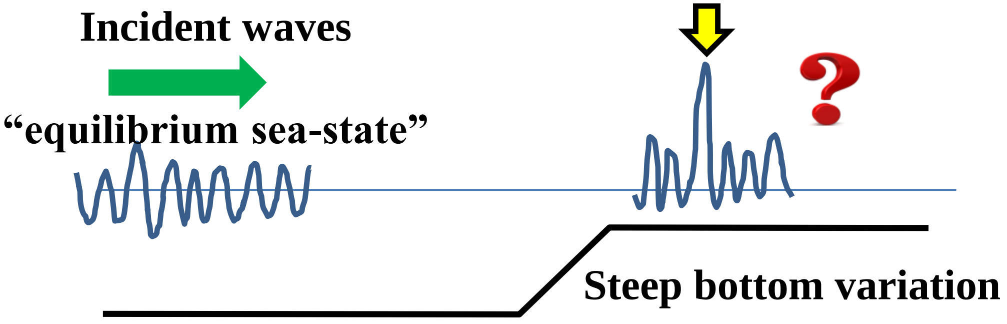
1. Geometrical focusing due wave propagation direction or uneven bottom.
2. Space-time focusing due to dispersion.
3. Wave-current interaction due to reflection and refraction.

Nonlinear

1. Modulational instability due to nonlinear four-wave interactions.
2. **Wave-bottom interaction due to the enhancement of bound harmonics.**
3. Space-time focusing due to nonlinear dispersion in shallow water depth.

Non-equilibrium + Non-Gaussian + Highly nonlinear
=> Enhanced occurrence of extreme waves?

Nonlinear wave-bottom interaction



Experiments – Selected experimental case

Irregular wave experiment: **Run 3 in Trulsen *et al.* (2020)**

Run	T_p (s)	γ	Deeper region					Shallower region				
			h_1 (m)	μ_1	H_{m_0} (m)	ϵ_1	Ur_1	h_2 (m)	μ_2	H_{m_0} (m)	ϵ_2	Ur_2
3	1.1	3.3	0.53	1.85	0.025	0.031	0.0049	0.11	0.64	0.025	0.052	0.1918

TABLE 1. Key parameters of the experimental case reported as Run 3 in Trulsen *et al.* (2020).

relative water depth $\mu = k_p h$

steepness $\epsilon = k_p a_c$

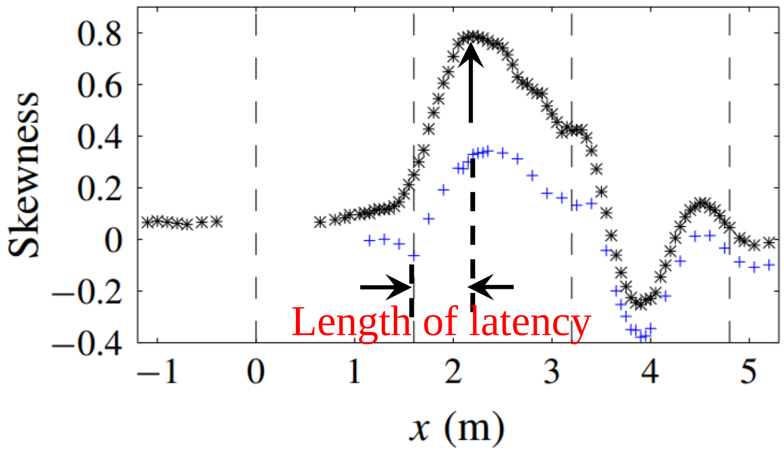
Ursell number $U_r = \epsilon / \mu^3$

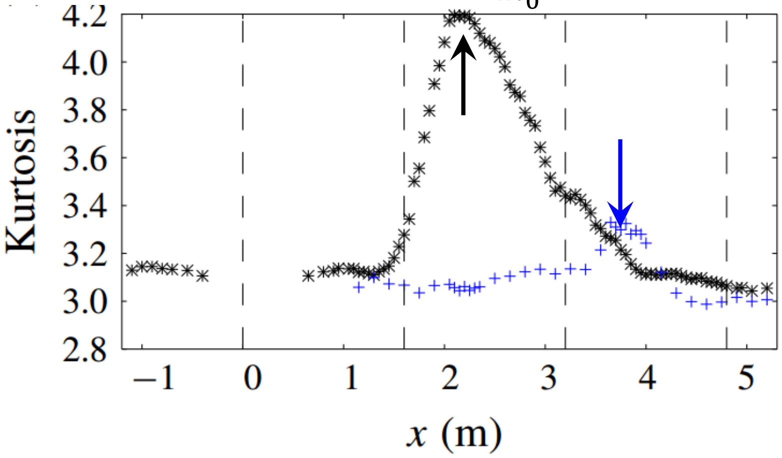
$$a_c = \sqrt{2}\sigma$$

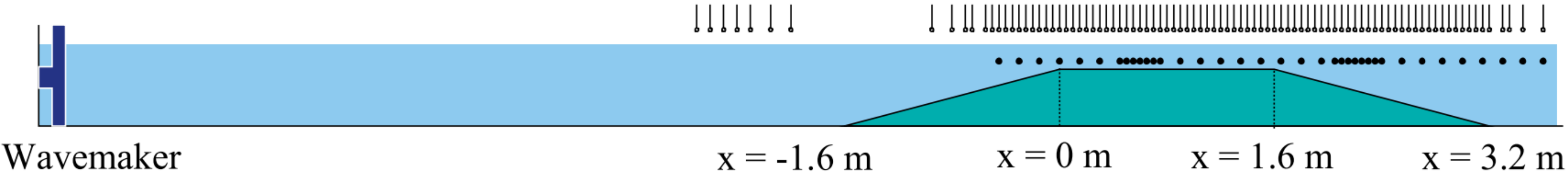
$$\sigma^2 = \langle (\eta - \langle \eta \rangle)^2 \rangle = m_0$$

$$\text{Skewness} = \frac{\langle (\eta - \langle \eta \rangle)^3 \rangle}{m_0^{3/2}}$$

$$\text{Kurtosis} = \frac{\langle (\eta - \langle \eta \rangle)^4 \rangle}{m_0^2}$$







Fully nonlinear and dispersive potential wave model: whispers3D

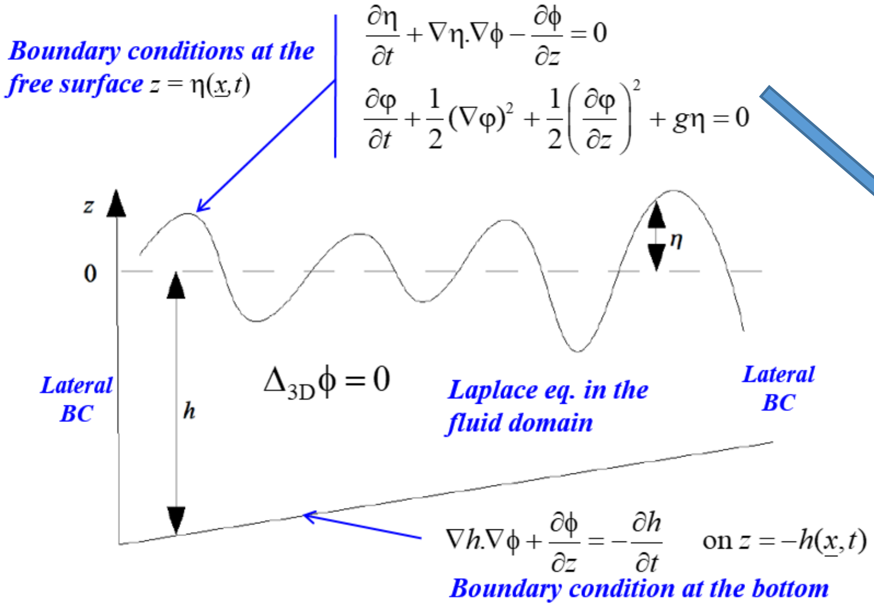
Main assumptions:

{H1} homogeneous fluid of constant density ρ

{H2} irrotational flow => wave potential $\phi(\underline{x},z,t)$: $\underline{u}(\underline{x},z,t) = \nabla\phi$ with $\underline{x} = (x,y)$

{H3} inviscid fluid

{H4} non-overturning waves => free surface elevation η is a single-valued function of $\underline{x}$.



Boundary conditions at the free surface $z = \eta(\underline{x},t)$

$$\frac{\partial\eta}{\partial t} + \nabla\eta.\nabla\phi - \frac{\partial\phi}{\partial z} = 0$$

$$\frac{\partial\varphi}{\partial t} + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial z}\right)^2 + g\eta = 0$$

Lateral BC

$$\Delta_{3D}\phi = 0$$

Laplace eq. in the fluid domain

Lateral BC

$$\nabla h.\nabla\phi + \frac{\partial\phi}{\partial z} = -\frac{\partial h}{\partial t} \quad \text{on } z = -h(\underline{x},t)$$

Boundary condition at the bottom

Introducing the free surface potential

$$\tilde{\varphi}(\underline{x},t) \equiv \phi(\underline{x}, z = \eta(\underline{x},t), t)$$

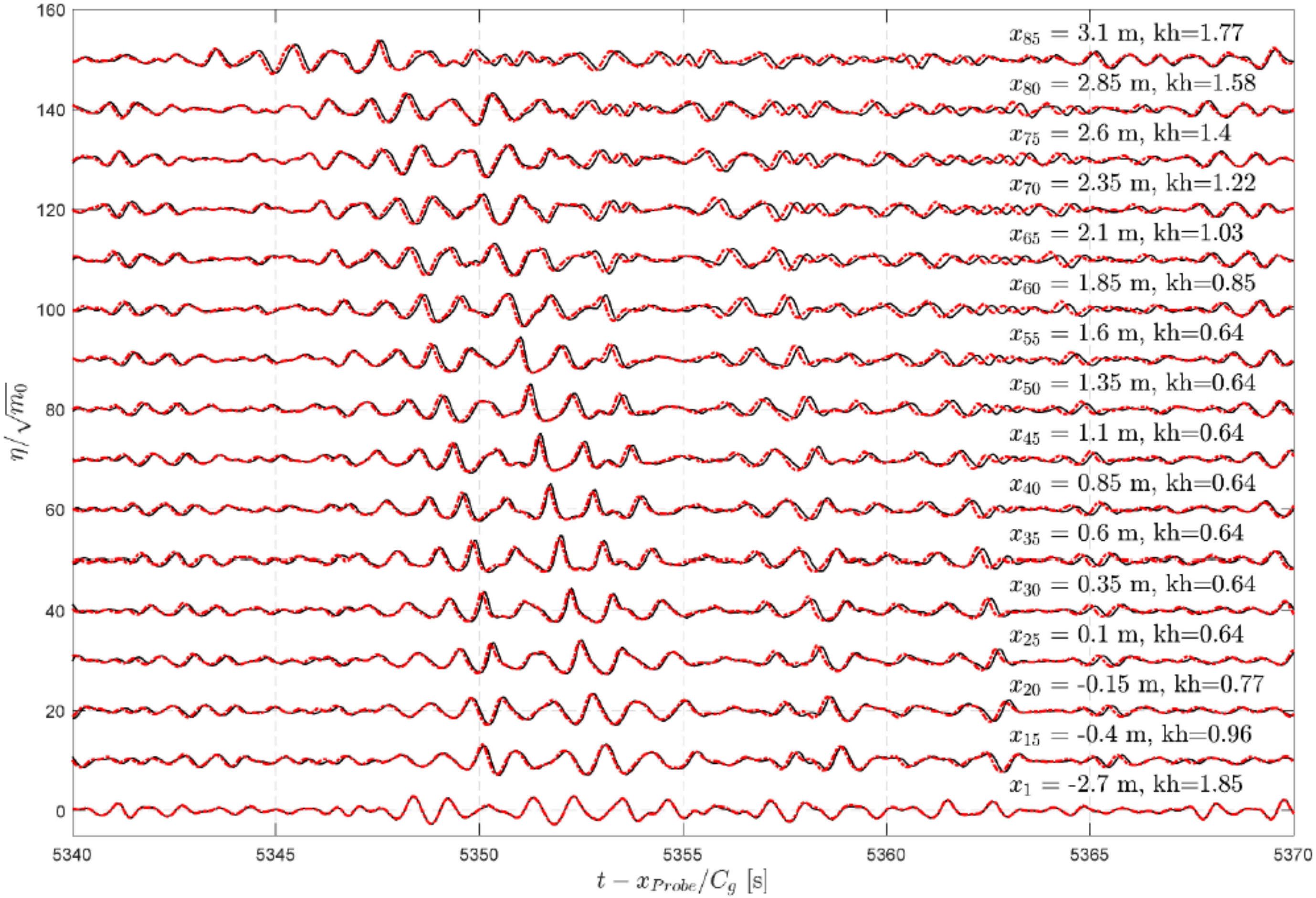
The 2 free-surface BC are written as the so-called **Zakharov equations** for $\eta(\underline{x},t)$ and $\tilde{\varphi}(\underline{x},t)$

$$\begin{cases} \dfrac{\partial\eta}{\partial t} = -\nabla\tilde{\phi}.\nabla\eta + \tilde{w}\left(1 + (\nabla\eta)^2\right) \\ \dfrac{\partial\tilde{\phi}}{\partial t} = -g\eta - \dfrac{1}{2}\left(\nabla\tilde{\phi}\right)^2 + \dfrac{1}{2}\tilde{w}^2\left(1 + (\nabla\eta)^2\right) \end{cases}$$

$\tilde{w}(\underline{x},t) = \dfrac{\partial\phi}{\partial z}(z = \eta)$ vertical velocity at the free surface.

- Spectral representation of the potential in the vertical **=> Whispers3D** code
  Yates & Benoit (2015); Raoult *et al.* (2016); Simon *et al.* (2019); Zhang et al. (2019).

Validation of wave model with Run 3 in TRJR2020



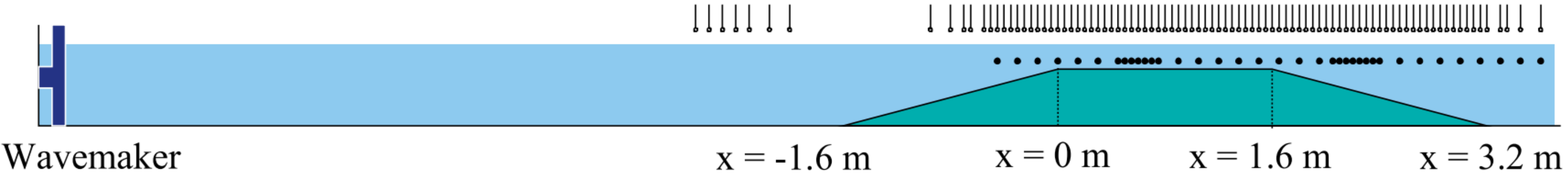
Comparison of the free surface elevation at 16 positions.

Snapshot after running simulations for nearly 90 min of physical time (i.e. after about 4,900 waves).

Black lines: measurements

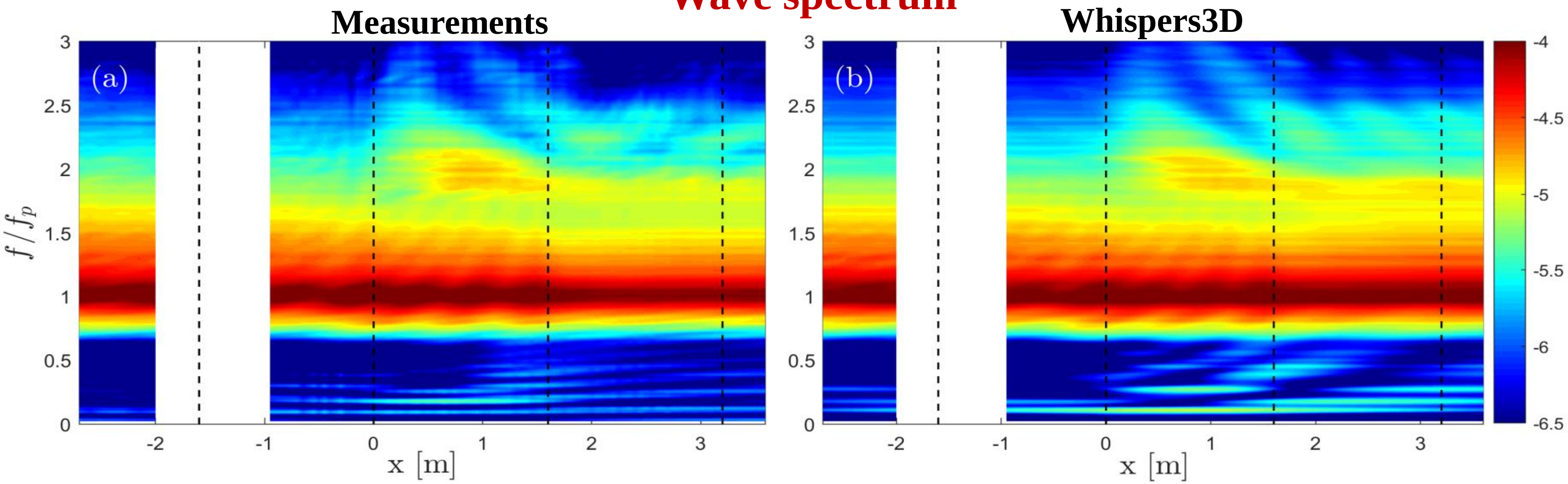
Red lines: simulations

Excellent agreement with measurements with nearly no phase lag!

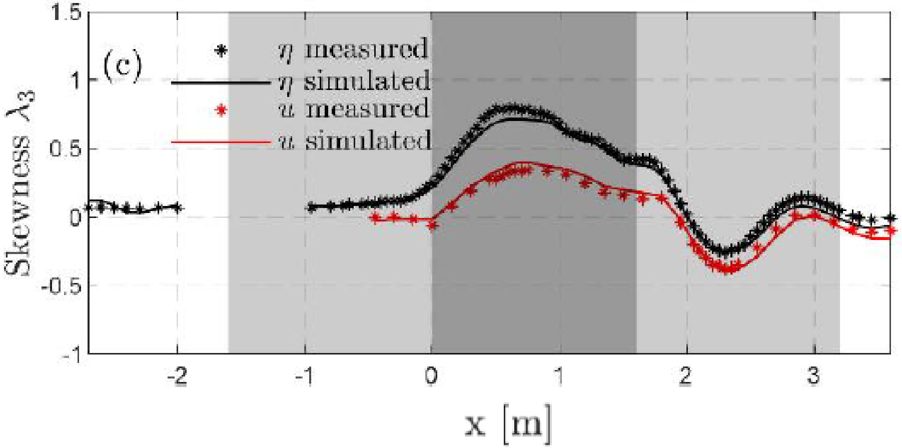


Validation of wave model with Run 3 in TRJR2020

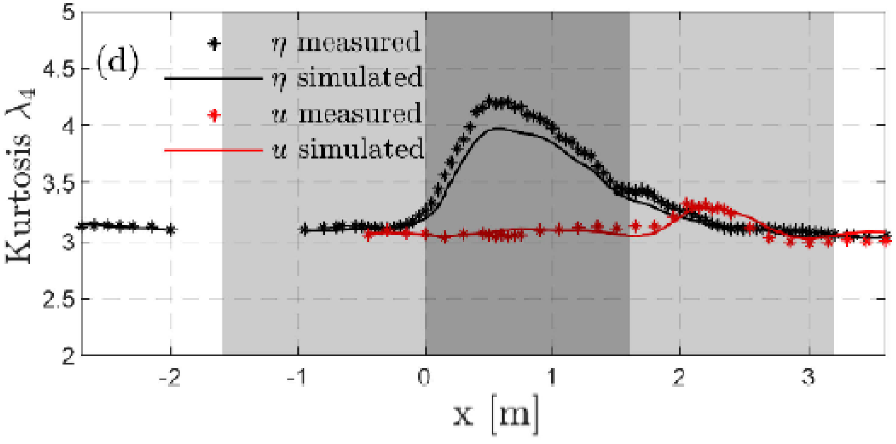
Wave spectrum



$$\text{Skewness} = \frac{\langle(\eta-\langle\eta\rangle)^3\rangle}{m_0^{3/2}}$$

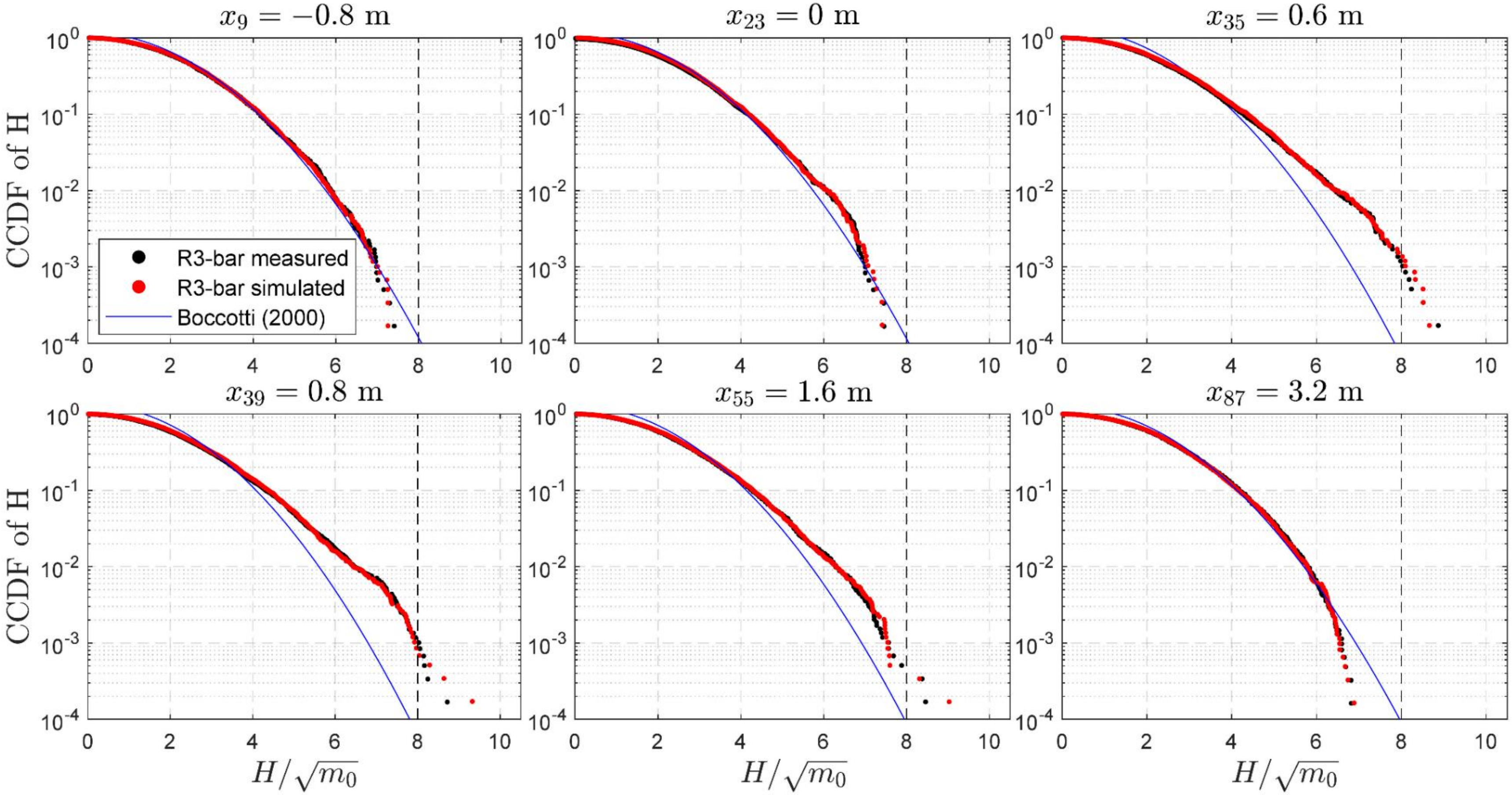


$$\text{Kurtosis} = \frac{\langle(\eta-\langle\eta\rangle)^4\rangle}{m_0^{2}}$$

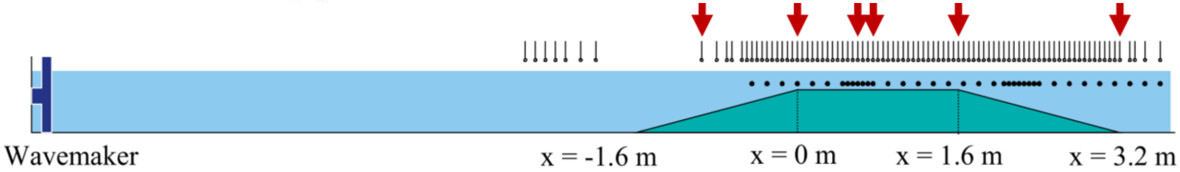


Validation – Statistical distribution of wave height

- **Separate individual waves with a zero-crossing method => wave heights *H***
- **Build the empirical Complementary Cumulative Distribution Function (CCDF) of *H***
- **Compare with existing distributions in finite water depth, here with Boccotti (2000) model.**



Vertical dash line $H = 8\sqrt{m_0} = 2H_{m_0}$
usual freak wave criterion

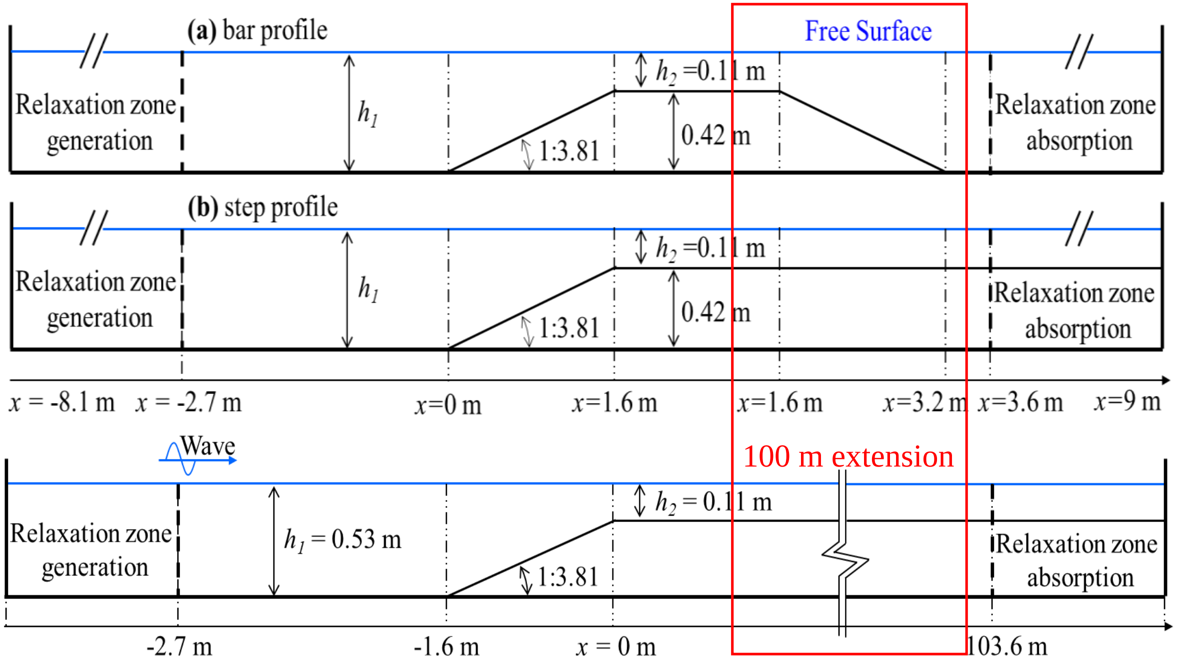


Simulation set 2 - Motivation for new tests

Questions: In which area does the depth-variation induced **Non-Equilibrium Dynamics (NED)** exist and influence shallow water waves?

No large scale study yet!

Motivation: Determining the range of out-of-equilibrium sea-states would help to evaluate the risk of freak waves



In Trulsen *et al.* 2020, JFM

In Zhang and Benoit 2021, JFM

In Zhang et al. (2022), Coastal Eng.

Simulation set 2 – Configurations of new tests

Summary of wave field parameters

Case	H_s [m]	L [m]	Deeper/shallower area μ	ε	Ur
1	0.030	103.6	1.85/0.64	0.037/0.062	0.0058/0.232
2	0.010	43.6	1.85/0.64	0.012/0.020	0.0019/0.077
3	0.015	43.6	1.85/0.64	0.019/0.031	0.0029/0.116
4	0.020	43.6	1.85/0.64	0.025/0.041	0.0039/0.155
5	0.025	43.6	1.85/0.64	0.031/0.052	0.0049/0.194
6	0.030	43.6	1.85/0.64	0.037/0.062	0.0058/0.232
7	0.035	43.6	1.85/0.64	0.043/0.073	0.0068/0.271

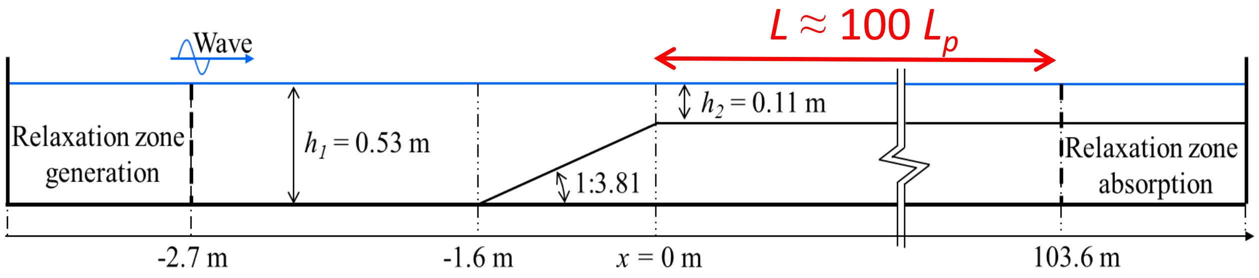
[a] For all cases, $T_p = 1.1$ s, $\gamma = 3.3$, deeper region depth $h_1 = 0.53$ m, shallower region depth $h_2 = 0.11$ m.

7 cases simulated.

Each case comprises 10 runs with different sets of random phases.

500 s duration for each run

=> about 4,500 waves in total



relative water depth $\mu = k_p h$

steepness $\epsilon = k_p a_c$

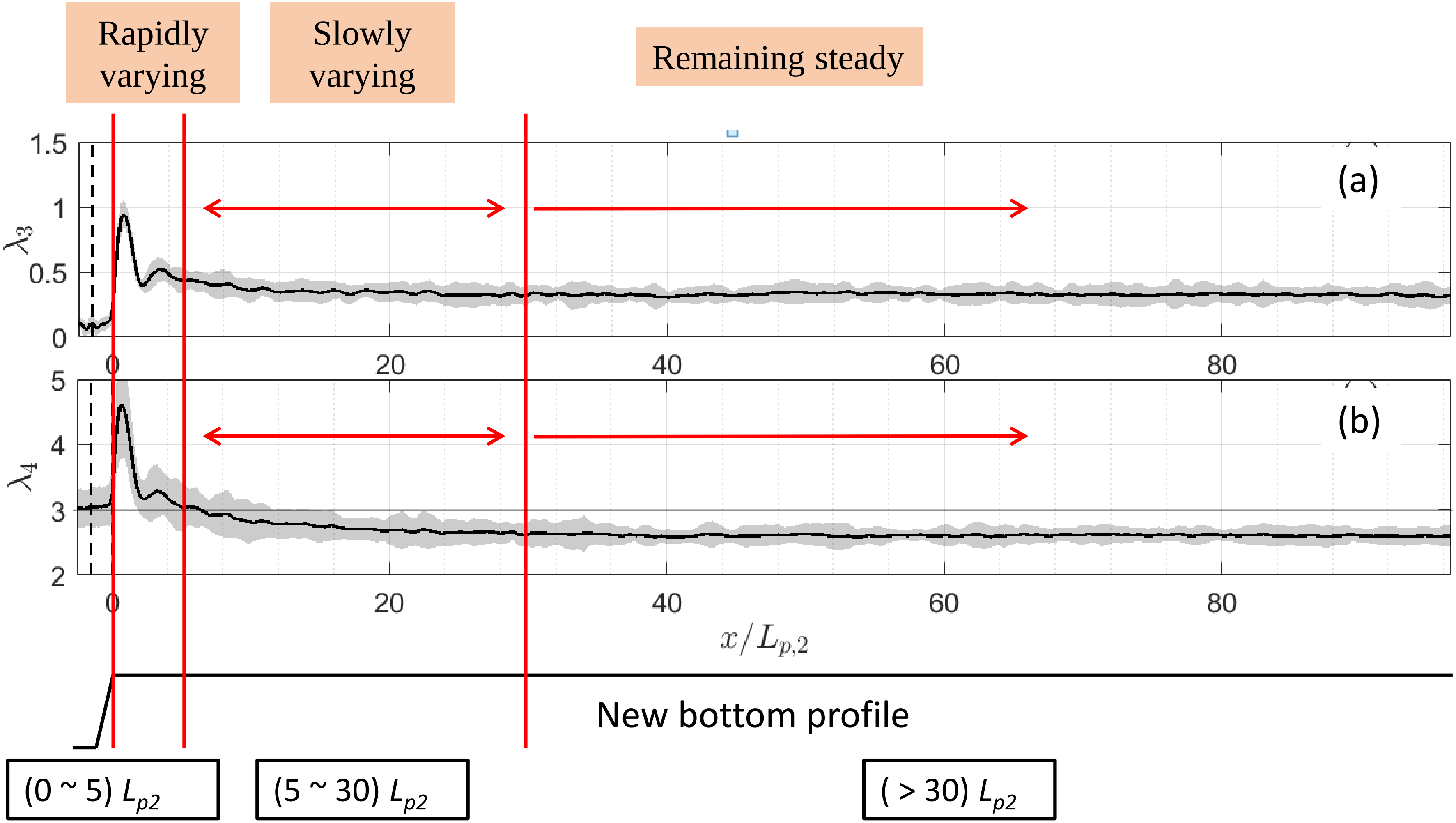
Ursell number $U_r = \epsilon / \mu^3$

$a_c = \sqrt{2}\sigma$

$\sigma^2 = \langle (\eta - \langle \eta \rangle)^2 \rangle = m_0$

Simulation set 2 – Statistical parameters

Case 1 (H_S = 0.03 m)



Rapidly varying

Slowly varying

Remaining steady

New bottom profile

(0 ~ 5) L_{p2}

(5 ~ 30) L_{p2}

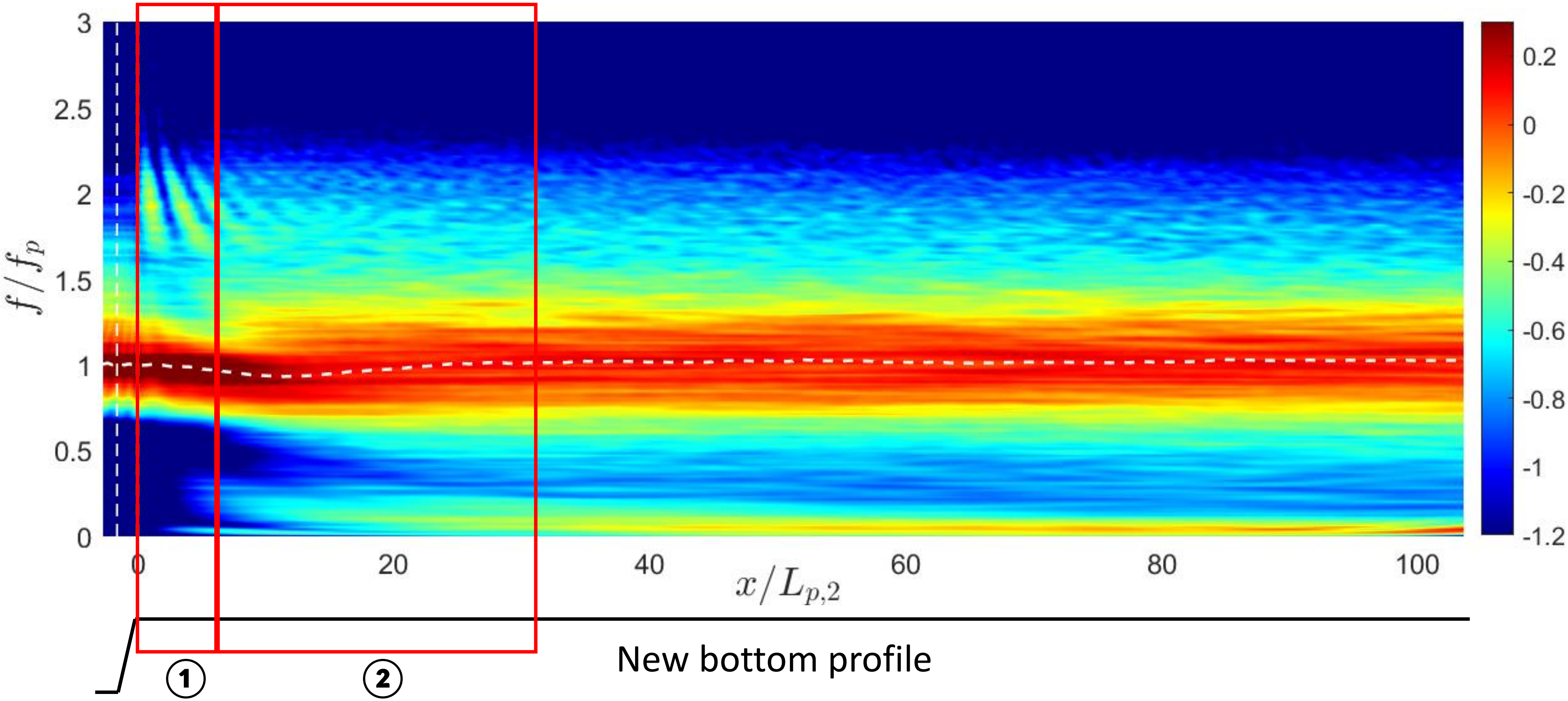
(> 30) L_{p2}

Simulation set 2 – Wave spectrum

Wave non-equilibrium dynamics induced by significant depth change has two phases:

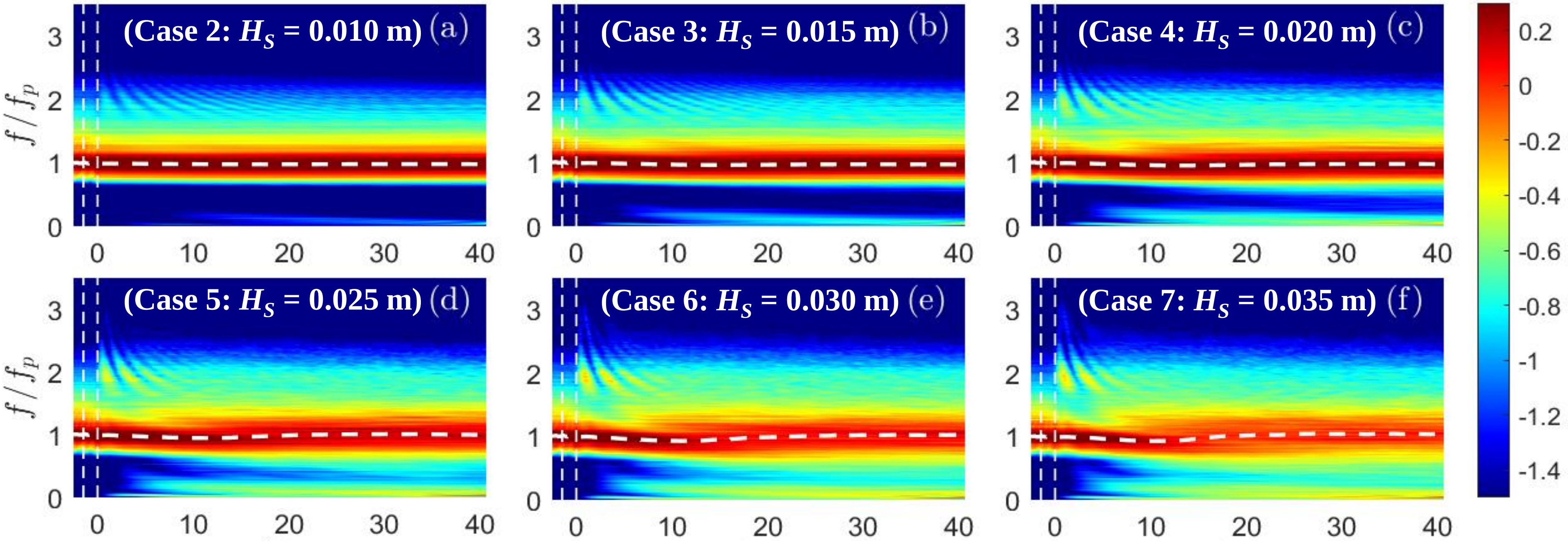
① short scale process (**intensified** freak wave probability + 2nd order harmonics);

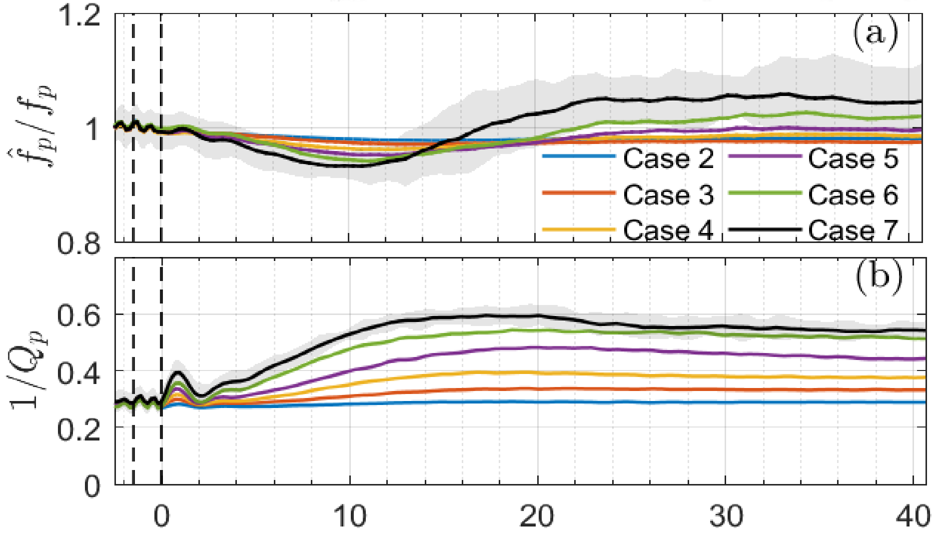
② Long scale process (**reduced** freak wave probability + broad band spectrum).

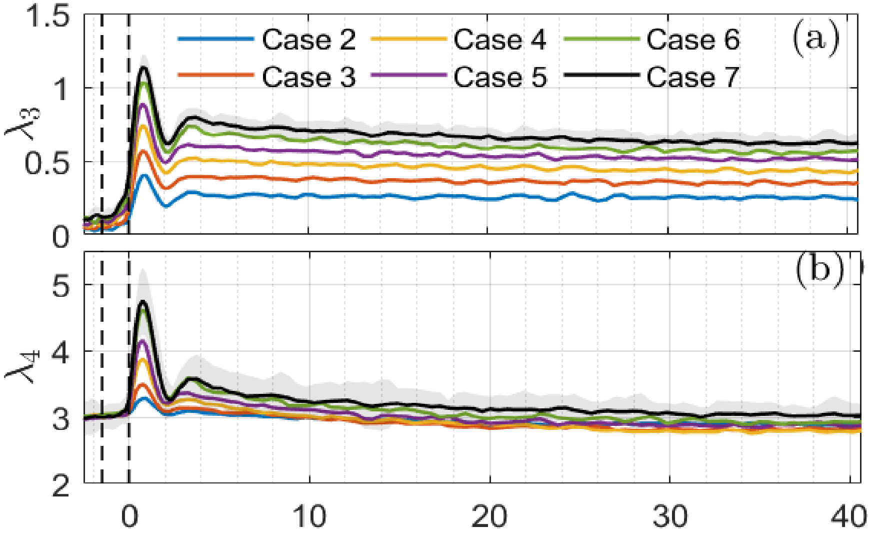


New bottom profile

Simulation set 2 – Effect of incident wave height

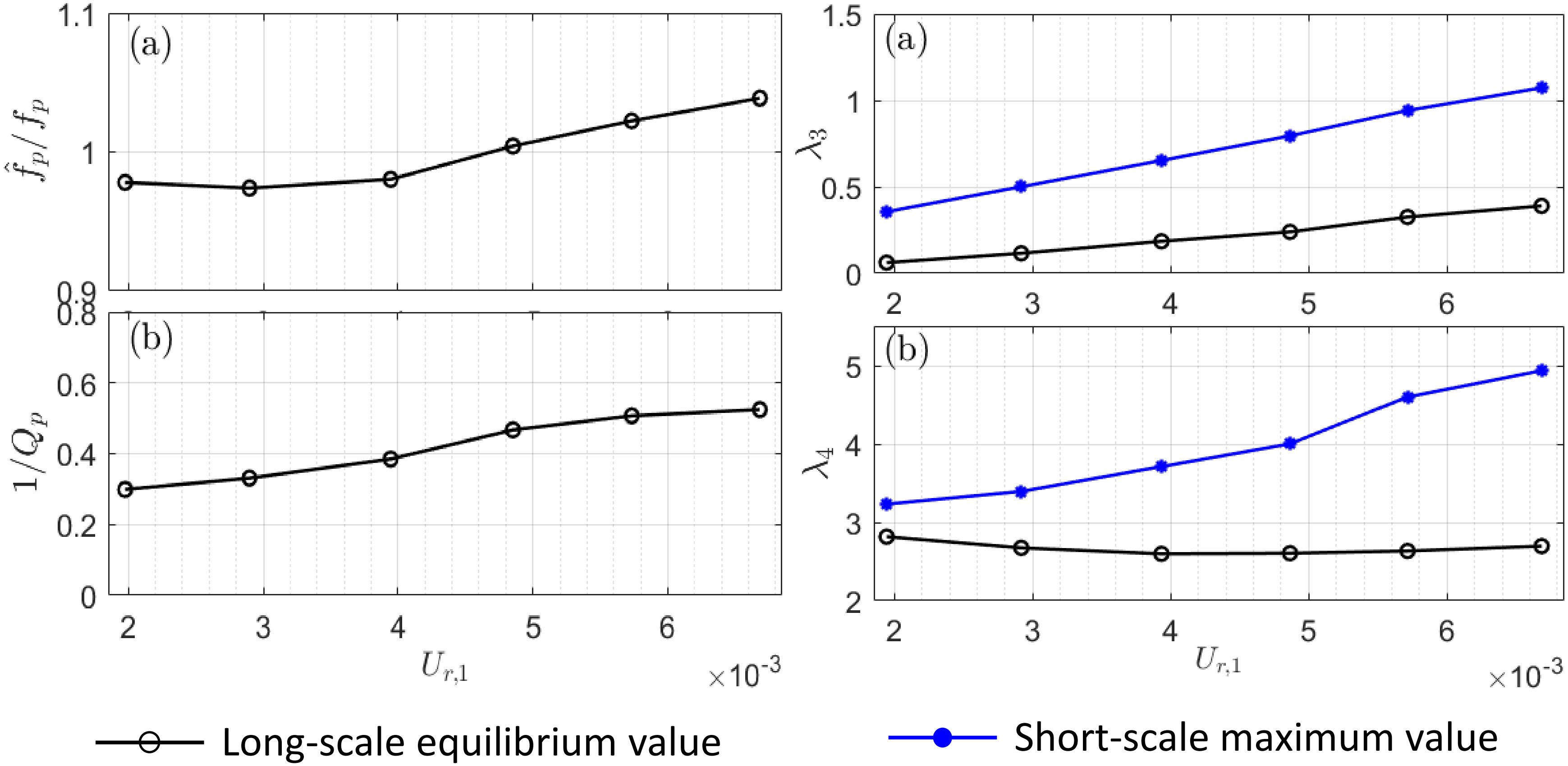






Simulation set 2 – Effect of incident wave height

Cases 2-7 : Effects of nonlinearity on spectral and statistical properties



Effect of with incident nonlinearity of kurtosis:

- the maximum value in the short scale increases (higher freak wave occurrence prob.),
- but the equilibrium level of does **NOT** increase.

Conclusions

- Performance of **whispers3D** model was proven excellent in the deterministic simulation of wave trains, for all aspects examined (spectra, statistics, bi-spectra,...), with ability to capture nonlinear wave-wave and wave-bottom interactions.

- Significant depth changes can result in **non-equilibrium dynamics (NED)** under certain circumstances (kh_{shallow} >1.3).

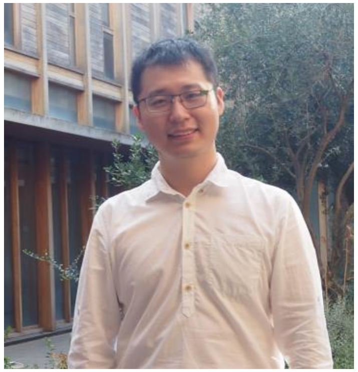
- The NED takes effects in two spatial scales, a the short scale O(L_p) and a relatively long scale O(10~10^2 L_p).

- In the short scale, the out-of-equilibrium sea-states are characterized by local enhancements of statistical parameters, transient 2nd and higher harmonics, and intensified freak wave probability.

- In the long scale, the statistics changes mildly whereas the spectral shape undergoes strong modulation (broadening around spectral peak), and reduced freak wave probability.

- After re-establishing the new shallow water equilibrium, freak waves are less likely to happen. A “safe zone” is expected, independent of the incident sea-state nonlinearity.

Thanks for your attention!



Jie Zhang

Related publications:

[1] Zhang, J., Benoit, M., Kimmoun, O., Chabchoub, A., Hsu, H.-C., 2019. Statistics of extreme waves in coastal waters: large scale experiments and advanced numerical simulations. **Fluids,** 4, 99.

[2] Zhang, J., Benoit, M., 2021. Wave–bottom interaction and extreme wave statistics due to shoaling and de-shoaling of irregular long-crested wave trains over steep seabed changes. **Journal of Fluid Mechanics,** 912, A28.

[3] Zhang J., Benoit M., Ma Y. (2022) Equilibration process of out-of-equilibrium sea-states induced by strong depth variation: Evolution of coastal wave spectrum and representative parameters. **Coastal Engineering**, 174, 104099.

[4] Zhang J. (2020) Wave-seabed interaction and mechanisms of freak wave formation in coastal zones. PhD thesis, Ecole Centrale Marseille & Irphé. Marseille (France) [http://theses.fr/2020ECDM0003]